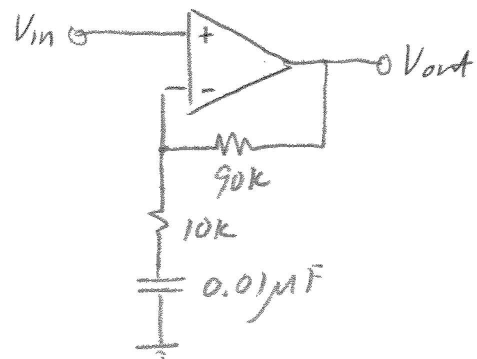
PROBLEM #1



(a) Transfer function

$V_- = V_+ = V_{in}$

Node V_-: $\dfrac{V_{in} - V_{out}}{90k} + \dfrac{V_{in} - 0}{10k + \frac{1}{s10^{-8}}} = 0$

$\dfrac{V_{out}}{V_{in}} = 1 + \dfrac{90k}{10k + \frac{1}{s10^{-8}}}$ ← could go directly to this using $1 + \frac{Z_2}{Z_1}$

$= \dfrac{10k + \frac{1}{s10^{-8}} + 90k}{10k + \frac{1}{s10^{-8}}} = \dfrac{s10^{-3} + 1}{s10^{-4} + 1} = \dfrac{10s + 10^4}{s + 10^4}$

$\boxed{\dfrac{V_{out}}{V_{in}}(s) = \dfrac{10(s+10^3)}{s+10^4}}$

Bode Plots next page.

(b) Gain at DC: let $s \to 0$ $\dfrac{V_{out}}{V_{in}} = \dfrac{10(0+10^3)}{0+10^4}$ $\boxed{\dfrac{V_{out}}{V_{in}} = 1}$

(c) Impulse response

$h(t) = \mathcal{L}^{-1}\left\{\dfrac{10(s+10^3)}{s+10^4}\right\} = \mathcal{L}^{-1}\left\{\dfrac{10(s+10^4)}{s+10^4} - \dfrac{9\times10^4}{s+10^4}\right\} = \mathcal{L}^{-1}\left\{10 - \dfrac{9\times10^4}{s+10^4}\right\}$

$\boxed{h(t) = 10\delta(t) - 9\times10^4 e^{-10^4 t}u(t)}$

(d) Output if input is $2000t\,u(t)$.

$V_{in}(s) = \dfrac{2000}{s^2}$ $V_{out}(s) = \dfrac{V_{out}(s)}{V_{in}(s)} V_{in}(s) = \dfrac{2\times10^4(s+10^3)}{s^2(s+10^4)}$

$V_{out}(s) = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+10^4}$

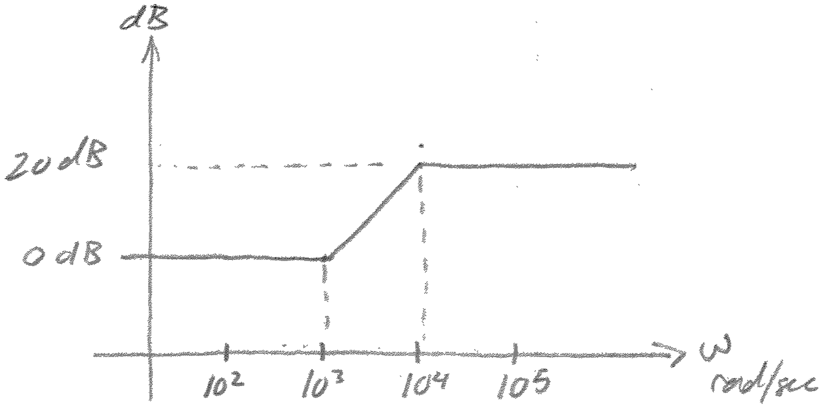
$A(s+10^4) + Bs(s+10^4) + Cs^2 = 2\times10^4 s + 2\times10^7$

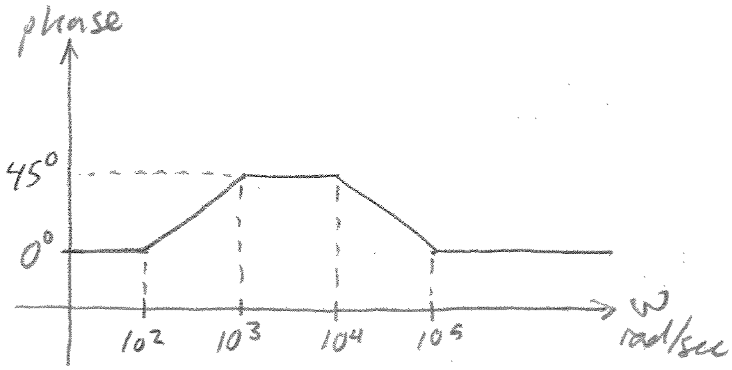
$(B+C)s^2 + (A+10^4B)s + 10^4A = 2\times10^7 \Rightarrow A = 2000,\ B = 1.8,\ C = -1.8$

$V_{out}(s) = \dfrac{2000}{s^2} + \dfrac{1.8}{s} - \dfrac{1.8}{s+10^4}$

$\boxed{V_{out}(t) = \left[2000t + 1.8 - 1.8e^{-10^4 t}\right]u(t)}$

PROBLEM #1 continued





PROBLEM #2

(a) $GBW \cong A_0 \omega_0 = 5\times10^7 \text{ rad/sec} = 7.96 \text{ MHz} \approx 8 \text{ MHz}$

If $G=40$, $BW = \dfrac{8 \text{ MHz}}{40} = \boxed{200 \text{ kHz}}$

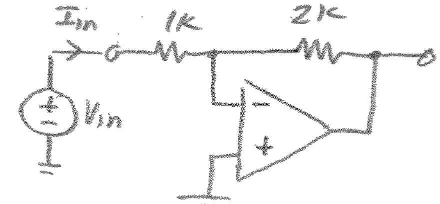
(b) If $G=4$, $BW = \dfrac{8 \text{ MHz}}{40} = \boxed{2 \text{ MHz}}$

(c) If $BW = 100 \text{ kHz}$, $G = \dfrac{8 \text{ MHz}}{100 \text{ kHz}} = \boxed{80}$

(d, e) Unity gain freq $= GBW = \boxed{8 \text{ MHz}}$

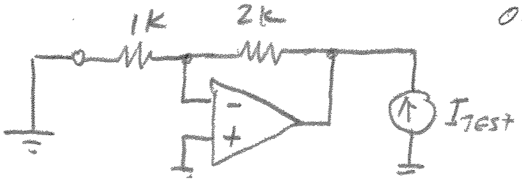
PROBLEM #3

(a) Ideal Case



$$G = -\frac{R_2}{R_1} = -\frac{2k}{1k} \Rightarrow \boxed{G = -2}$$

$$I_{in} = \frac{V_{in} - V_-}{1k} = \frac{V_{in} - 0}{1k} \qquad R_{in} = \frac{V_{in}}{I_{in}} \Rightarrow \boxed{R_{in} = 1\,k\Omega}$$

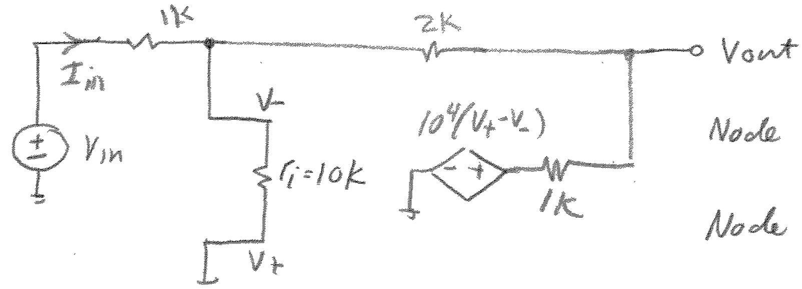


output impedance

$$V_- = 0 \text{ so: } \frac{0-0}{1k} + \frac{0 - V_{out}}{2k} = 0 \Rightarrow V_{out} = 0.$$

$$R_{out} = \frac{V_{out}}{I_{Test}} \Rightarrow \boxed{R_{out} = 0}$$

(b)



Node V_-: $$\frac{V_- - V_{in}}{1k} + \frac{V_- - 0}{10k} + \frac{V_- - V_{out}}{2k} = 0$$

Node V_{out}: $$\frac{V_{out} - V_-}{2k} + \frac{V_{out} - 10^4(0 - V_-)}{1k} = 0$$

Simplify first eqn: $$V_-\left(\frac{1}{1k} + \frac{1}{10k} + \frac{1}{2k}\right) = \frac{V_{in}}{1k} + \frac{V_{out}}{2k}$$

$$\Rightarrow V_-(10 + 1 + 5) = 10 V_{in} + 5 V_{out} \Rightarrow V_- = \frac{10}{16} V_{in} + \frac{5}{16} V_{out}$$

Substitute into 2nd: $$\frac{V_{out} - \left(\frac{10}{16} V_{in} + \frac{5}{16} V_{out}\right)}{2k} + \frac{V_{out} - 10^4\left(\frac{10}{16} V_{in} + \frac{5}{16} V_{out}\right)}{1k} = 0$$

$$V_{out}\left(\frac{1}{2k} - \frac{5}{16(2k)} + \frac{1}{1k} - \frac{10^4(5)}{16(1k)}\right) = V_{in}\left(\frac{10}{16(2k)} + \frac{10^4(10)}{16(1k)}\right)$$

$$V_{out}(-3.12565625) = V_{in}(6.2503125) \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = -1.99968}$$

(c) Input resistance: Use same circuit

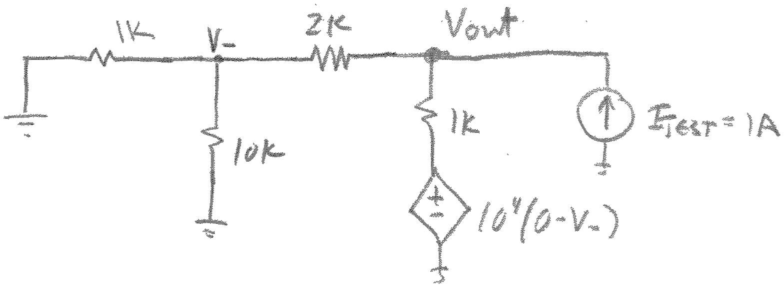
$$I_{in} = \frac{V_{in} - V_-}{1k} = \frac{V_{in} - \left(\frac{10}{16} V_{in} + \frac{5}{16} V_{out}\right)}{1k} = \frac{V_{in} - \left(\frac{10}{16} V_{in} + \frac{5}{16}(-1.99968) V_{in}\right)}{1k}$$

$$I_{in} = \frac{0.9999}{1k} V_{in} \qquad R_{in} = \frac{V_{in}}{I_{in}} = \frac{1k}{0.9999} \qquad \boxed{R_{in} = 1.0001\,k\Omega}$$

(You could let $V_{in} = 1V$ if you wanted - same answer)

PROBLEM #3 continued

(d) Output resistance



$$\frac{V_- - 0}{1k} + \frac{V_- - 0}{10k} + \frac{V_- - V_{out}}{2k} = 0.$$

$$\frac{V_{out} - V_-}{2k} + \frac{V_{out} + 10^4 V_-}{1k} - 1 = 0.$$

From first equation $V_- = 0.3125\,V_{out}$. Substitute into 2nd equation

$$\frac{V_{out} - 0.3125 V_{out}}{2k} + \frac{V_{out} + 10^4(0.3125 V_{out})}{1k} = 1 \Rightarrow V_{out} = 0.3199$$

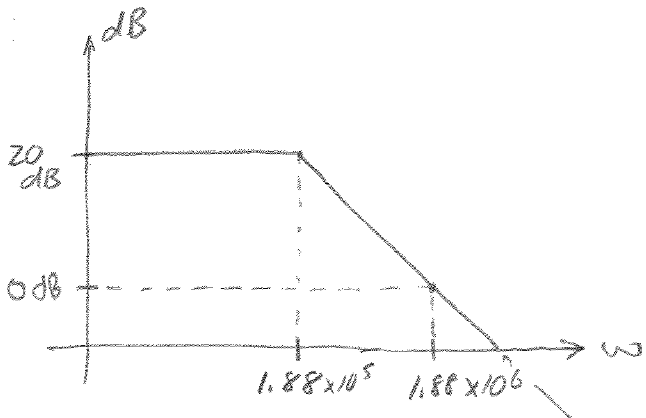
$$R_{out} = \frac{V_{out}}{I_{TEST}} \Rightarrow \boxed{R_{out} = 0.3199\,\Omega}$$

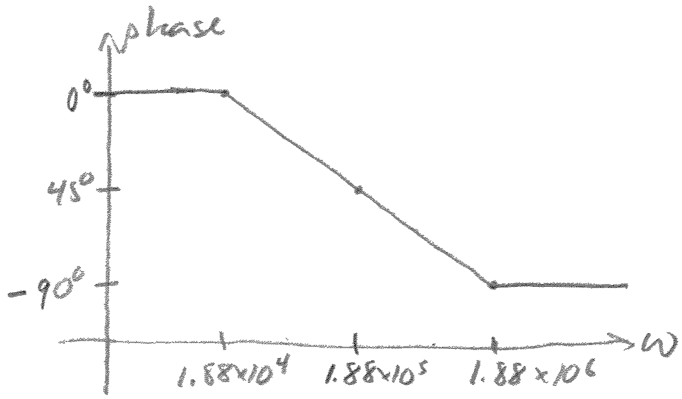
PROBLEM #4

Unity gain frequency $= 3\,\text{MHz} = 1.88\,\text{Mrad/sec.} = \omega_{0dB}$

(a) $H(s) \approx \dfrac{\omega_{0dB}}{s + \dfrac{\omega_{0dB}}{G}}$ where G is ideal DC gain.

$$\boxed{H(s) = \frac{1.88 \times 10^6}{s + 1.88 \times 10^5}}$$



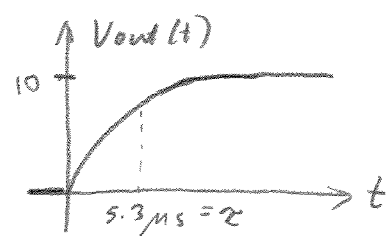


(b) Step response: $V_{in}(t) = u(t)$ $V_{in}(s) = \frac{1}{s}$.

$$V_{out}(s) = \frac{V_{out}}{V_{in}}(s)\,V_{in}(s) = H(s)V_{in}(s) = \frac{1.88\times10^6}{s(s+1.88\times10^5)} = \frac{A}{s} + \frac{B}{s+1.88\times10^5}$$

$$A = 10,\ B = -10. \Rightarrow V_{out}(s) = \frac{10}{s} - \frac{10}{s+1.88\times10^5}$$

$$\boxed{V_{out}(t) = \left(10 - 10e^{-1.88\times10^5 t}\right)u(t)}$$



PROBLEM #4 continued

(c) Sinusoidal steady state $s = j\omega$, use phasor analysis

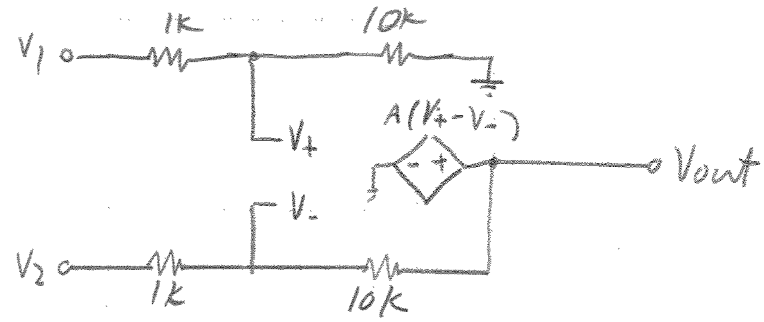
$V_{in}(t) = 3\cos(4\times10^6 t) \Rightarrow V_{in} = 3\angle 0^\circ, \quad s = j\omega = j4\times10^6$

$$\frac{V_{out}}{V_{in}} = \frac{1.88\times10^6}{j4\times10^6 + 1.88\times10^5} = 0.4695\angle -87.3^\circ$$

$$V_{out} = \frac{V_{out}}{V_{in}} V_{in} = (0.4695\angle -87.3^\circ)(3\angle 0^\circ) = 1.408\angle -87.3^\circ$$

$$\boxed{V_{out}(t) = 1.408\cos(4\times10^6 t - 87.3^\circ) \text{ Volts}}$$

PROBLEM #5



Node V_+: $\frac{V_+ - V_1}{1k} + \frac{V_+ - 0}{10k} = 0$

$\Rightarrow V_+ = V_1 \frac{10}{11}$

Node V_-: $\frac{V_- - V_2}{1k} + \frac{V_- - A(V_+ - V_-)}{10k} = 0$

substitute for V_+: $\frac{V_- - V_2}{1k} + \frac{V_- - A(\frac{10}{11}V_1 - V_-)}{10k} = 0.$

$$V_-\left(\frac{1}{1k} + \frac{1}{10k} + \frac{A}{10k}\right) = \frac{A(\frac{10}{11})V_1}{10k} + \frac{V_2}{1k} \Rightarrow V_-(A+11) = A\tfrac{10}{11}V_1 + 10V_2$$

$$V_- = \frac{A(10/11)V_1 + 10V_2}{A+11} = \frac{10AV_1 + 110V_2}{11A + 121}$$

Then $V_{out} = A(V_+ - V_-)$

$$V_{out} = A\left(V_1\frac{10}{11} - \frac{10AV_1 + 110V_2}{11A+121}\right) = A\frac{V_1(10)(A+11) - (10AV_1 + 110V_2)}{11A+121}$$

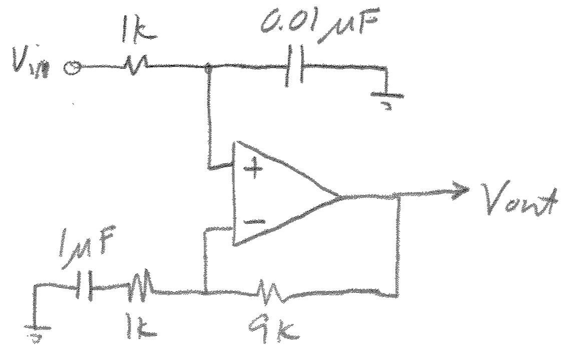
$$= A\frac{110(V_1 - V_2)}{11A+121} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{10A(V_1 - V_2)}{A+11}}$$

(b) $\boxed{\lim_{A\to\infty} \frac{V_{out}}{V_{in}} = 10(V_1 - V_2)}$

(c) If $V_1 = 1V$, $V_2 = 0V$, Ideal Value is $10V$. 1% diff $= 9.9$ Volts.

$$9.9 = \frac{10A(1-0)}{A+11} \qquad \boxed{A \geq 1089}$$

PROBLEM #6



(b) $\frac{V_+ - V_{in}}{1k} + \frac{V_+ - 0}{1/s10^{-8}} = 0$ (Node V_+)

or use voltage divider.

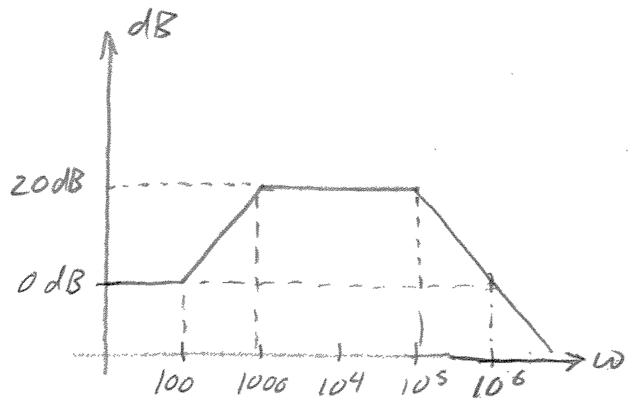
$$V_+ = V_{in}\,\frac{1/s10^{-8}}{1/s10^{-8} + 1k} = V_{in}\,\frac{10^5}{s+10^5}$$

Node V_-: $\frac{V_- - 0}{1k + \frac{1}{s10^{-6}}} + \frac{V_- - V_{out}}{9k} = 0 \Rightarrow V_{out} = V_-\,\frac{1k + \frac{1}{s10^{-6}} + 9k}{1k + \frac{1}{s10^{-6}}}$

$$V_{out} = V_-\,\frac{s10^{-2} + 1}{s10^{-3} + 1} = V_-\,\frac{10s + 1000}{s + 1000} = \frac{10(s+100)}{(s+1000)}\,V_-$$

Then use $V_- = V_+ = V_{in}\,\frac{10^5}{s+10^5}$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{10^6(s+100)}{(s+1000)(s+10^5)}}$$



(a) Let $s \to 0$.

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{10^6(100)}{(1000)(10^5)} = 1}$$

(d) If $V_{in} = 2u(t)$, $V_{in}(s) = \frac{2}{s}$

$$V_{out}(s) = \frac{V_{out}}{V_{in}}(s)\,V_{in}(s) = \frac{2\times10^6(s+100)}{s(s+1000)(s+10^5)}$$

$$V_{out}(s) = \frac{A}{s} + \frac{B}{s+1000} + \frac{C}{s+10^5} \rightarrow A = 2,\quad B = 18.18,\quad C = -20.1818$$

$$V_{out}(s) = \frac{2}{s} + \frac{18.18}{s+1000} - \frac{20.18}{s+10^5}$$

$$\boxed{V_{out}(t) = \left[2 + 18.18e^{-10^3 t} - 20.18e^{-10^5 t}\right]u(t)}$$